Government of **Western Australia**
**Curriculum Council**

WACE
Western Australian Certificate of Education

# Western Australian Certificate of Education Examination, 2010

## Question/Answer Booklet

# MATHEMATICS: SPECIALIST 3C/3D

## Section Two: Calculator-assumed

Please place your student identification label in this box

Student Number: In figures | | | | | | | | |

In words _____

_____

## Time allowed for this section

Reading time before commencing work: ten minutes
Working time for this section: one hundred minutes

## Materials required/recommended for this section

***To be provided by the supervisor***

This Question/Answer Booklet
Formula Sheet (retained from Section One)

***To be provided by the candidate***

Standard items: pens, pencils, pencil sharpener, eraser, correction fluid/tape, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators satisfying the conditions set by the Curriculum Council for this examination

## Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

## Structure of this paper

| Section | Number of questions available | Number of questions to be answered | Working time (minutes) | Marks available | Percentage of exam |
|---|---|---|---|---|---|
| Section One: Calculator-free | 7 | 7 | 50 | 40 | |
| Section Two: Calculator-assumed | 11 | 11 | 100 | 80 | |
| | | | **Total** | 120 | 100 |

## Instructions to candidates

1. The rules for the conduct of Western Australian external examinations are detailed in the *Year 12 Information Handbook 2010.* Sitting this examination implies that you agree to abide by these rules.

2. Answer the questions according to the following instructions.

   Section Two: Write answers in this Question/Answer Booklet. Answer **all** questions.

   **Show all your working clearly**. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.

   It is recommended that you **do not use pencil**, except in diagrams.

3. You must be careful to confine your responses to the specific questions asked and to follow any instructions that are specific to a particular question.

4. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
   - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
   - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question(s) that you are continuing to answer at the top of the page.



**See next page**

**Section Two: Calculator-assumed** **(80 Marks)**

This section has **11** questions. Answer **all** questions. Write your answers in the spaces provided.

Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.

- Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
- Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question(s) that you are continuing to answer at the top of the page.

Working time: 100 minutes.

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**Question 8** **(4 marks)**

Points $P$ and $Q$ have coordinates $(3, 1, -2)$ and $(4, 2, -1)$ respectively.

(a) Write a vector equation for the line passing through $P$ and $Q$. (2 marks)

(b) Show that the vector $2\boldsymbol{i} - \boldsymbol{j} - \boldsymbol{k}$ is perpendicular to the line through $P$ and $Q$. (1 mark)

(c) Write down a vector equation of the plane containing $P$ and $Q$ with $2\boldsymbol{i} - \boldsymbol{j} - \boldsymbol{k}$ as its normal vector. (1 mark)

**Question 9** **(5 marks)**

The vertices of the triangle $ABC$ have coordinates $A(2, 1, 4)$, $B(7, 1, 0)$ and $C(-7, 5, 2)$.

(a) Find the vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$. (2 marks)

(b) Find the size of $\angle BAC$, correct to the nearest degree. (1 mark)

(c) The point $D$ divides $\overrightarrow{AC}$ internally in the ratio 5:1. Find the vector $\overrightarrow{BD}$. (2 marks)

**Question 10** **(7 marks)**

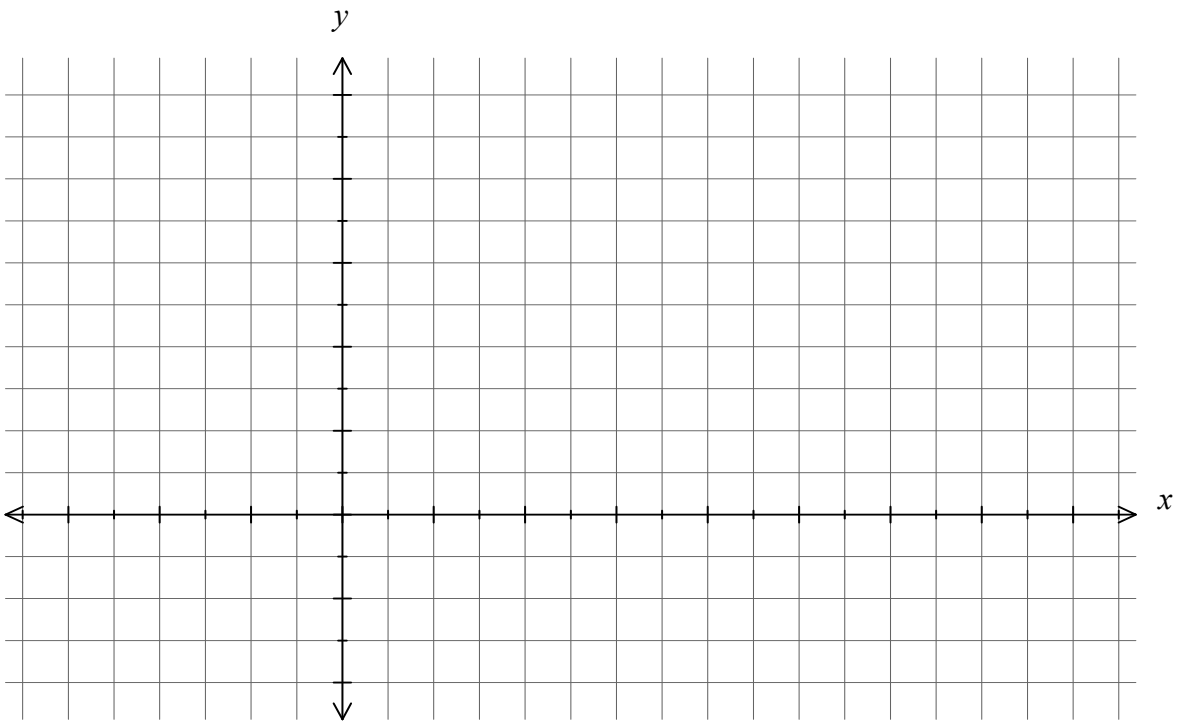
The function $y = f(x)$ is given by $f(x) = e^{ax-2}$, where $a$ is a constant.

When $y = f(x)$ and $y = f^{-1}(x)$ are plotted on the same set of axes, they intersect at a point where $x = 3$.

(a) Write down the exact value of $y$ at this point of intersection. (1 mark)

(b) Find the value of $a$ correct to 2 decimal places. (2 marks)

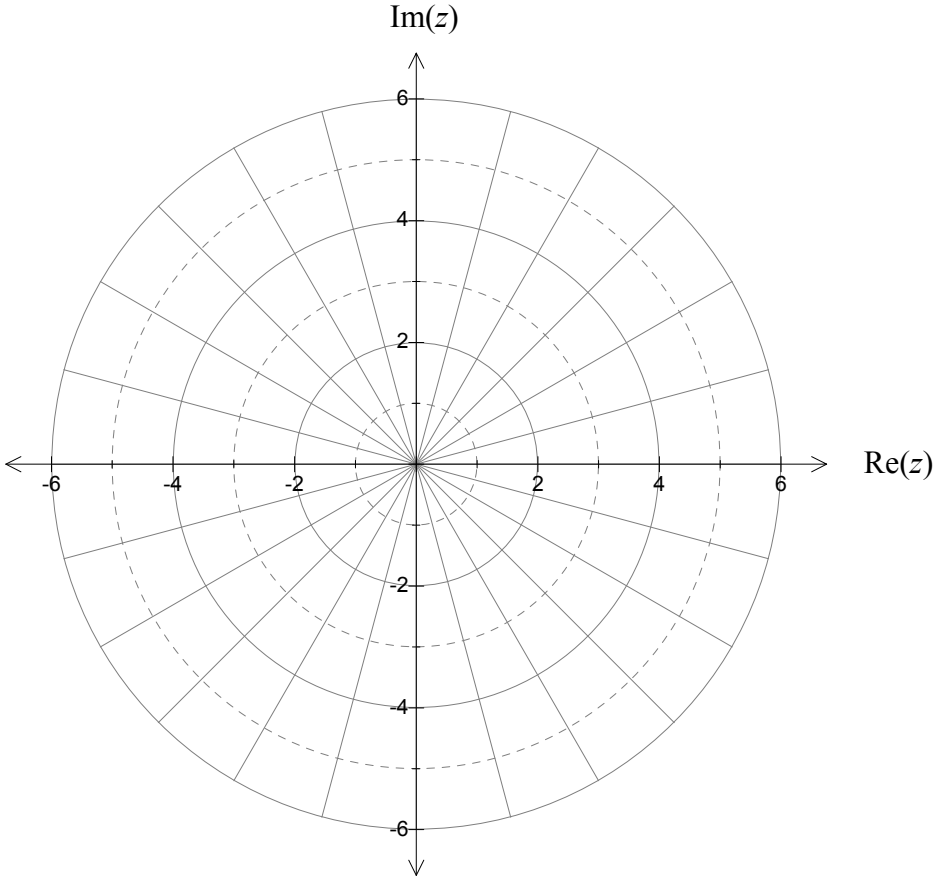
(c) Draw $y = f(x)$ and $y = f^{-1}(x)$ on the same set of axes, showing the coordinates of the point(s) of intersection. (4 marks)



**Question 11** **(9 marks)**

(a) Using your CAS calculator (or otherwise) find all solutions to $z^5 = \frac{3125}{2}\left(-\sqrt{3}+i\right)$ in exact polar form, where $z = r\left(\cos\theta + i\sin\theta\right)$, $-\pi < \theta \le \pi$ and $r \ge 0$. (5 marks)

(b) Draw the solutions from (a) on the complex plane below. Show all major features. (4 marks)

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**Question 12** **(6 marks)**

(a) Use the method of mathematical induction to prove that

$1^2+2^2+3^2+\ldots+n^2=\frac{1}{6}n(n+1)(2n+1)$ for $n\geq 1$. (5 marks)

(b) Hence evaluate $\lim_{n\to\infty}\left[\frac{1^2+2^2+3^2+\ldots+n^2}{n^3}\right]$. (1 mark)

**Question 13** **(9 marks)**

Bharat and Lena have been monitoring a small population of woylies introduced into a wildlife sanctuary. They have collected the following data on the females in the population:

| Age (years) | 0–1 | 1–2 | 2–3 | 3–4 | 4–5 |
|---|---|---|---|---|---|
| Initial population | 2 | 5 | 4 | 4 | 0 |
| Birth rate | 0 | 1.9 | 2.2 | 1.2 | 0.9 |
| Survival rate | 0.5 | 0.8 | 0.5 | 0.3 | 0 |

(a) Lena will now represent the above data as a Leslie Matrix to predict future populations. Write down such a matrix. (1 mark)

(b) Showing working to support your answer, find (to the nearest whole number) the total female population after:

(i) 1 year

(ii) 2 years

(iii) 5 years (3 marks)



**See next page**

(c) Lena must predict when the female population will be at least 10 times its original size. How soon should this happen? Justify your answer. (2 marks)

(d) After eight years, Bharat observes that the population is badly affected by disease and that the survival rates have decreased. The rate for 4–5 year-olds remains at 0 but all other rates have fallen to 0.2. If no action is taken, in which year will Bharat find that the female population has fallen below 100 again? Justify your answer. (3 marks)



See next page

**Question 14** **(10 marks)**

A ball oscillates on the end of a spring such that its vertical motion can be described by the solution of the differential equation, $\frac{d^2x}{dt^2} = -4x$, where $x(t)$ cm is the vertical displacement of the ball about the mean position and $t$ is measured in seconds.

If the amplitude of motion is 4 cm and when $t = 0,\ x = 0$ and $\frac{dx}{dt} > 0$:

(a) (i) Determine $x(t)$. (2 marks)

(ii) Find, exactly, the first two times $t > 0$ that the ball is 2 cm away from its mean position. (2 marks)

(b) For what fraction of time will the ball be at least 2 cm away from its mean position? (3 marks)

(c) How far does the ball travel in the first 10 seconds? (3 marks)

**Question 15** **(9 marks)**

Two rockets are fired from different positions at the same time. Rocket 1 leaves from position $-7\boldsymbol{i}+9\boldsymbol{j}-5\boldsymbol{k}$ km at a velocity of $5\boldsymbol{i}-4\boldsymbol{j}+2\boldsymbol{k}$ km/min and Rocket 2 leaves from position $-6\boldsymbol{i}-5\boldsymbol{j}+2\boldsymbol{k}$ km at a velocity of $9\boldsymbol{i}+6\boldsymbol{j}-3\boldsymbol{k}$ km/min. Each rocket leaves a trail of smoke and, although the rockets do not collide, their smoke trails do intersect.

(a) Find the coordinates of the point at which the smoke trails intersect. (4 marks)

(b) Determine the shortest distance of Rocket 1 from the smoke trail of Rocket 2, three minutes after firing. Give your answer to the nearest metre. (5 marks)



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**Question 16** **(7 marks)**

In a particular valley, only two types of plants grow: Blues and Reds. Today there are 130 hectares of Blue plants and 40 hectares of Red plants.

Due to a disease, that affects only Blue plants, the number of hectares of Blue plants, $B$, can be described by the differential equation $\frac{dB}{dt} = -0.15B$, where $t$ is measured in years.

(a) In how many years will the number of hectares of Blue plants halve? (3 marks)

(b) The number of hectares of Red plants, $R$, grows exponentially. Find the percentage growth rate in $R$ if, after seven years, the total number of hectares of Blue and Red plants is the same as today. (4 marks)



See next page

**Question 17** **(4 marks)**

(a) Write down the result (correct to five decimal places) of entering $i^i$ ($i$ to the power of $i$) into your CAS calculator. Comment on the answer you have recorded. (2 marks)

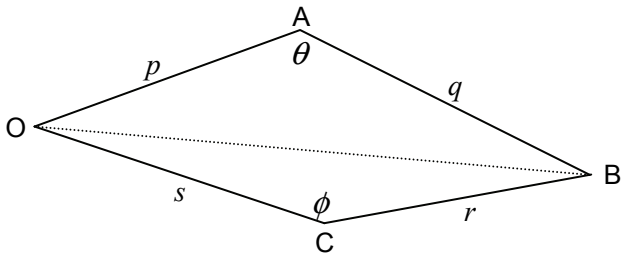
(b) Express your result for $z = i^i$ exactly in the form $z = a + bi$, where $a$ and $b$ are real. (2 marks)



**See next page**

**Question 18** **(10 marks)**

The quadrilateral OABC, shown below, has sides $p, q, r$ and $s$, each of fixed length. The opposite angles $\theta$ and $\phi$ are variable.




(a) The cosine rule can be used in each of the triangles above to define the length of OB. Using these expressions for the length of OB and implicit differentiation, show that $\frac{d\phi}{d\theta} = \frac{pq\sin\theta}{rs\sin\phi}$. (3 marks)

(b) If $A$ is the area of this quadrilaqteral, use calculus to show that $\frac{dA}{d\theta}=0$ when $\theta$ and $\phi$ are supplementary (i.e. $\theta+\phi=180°$). (7 marks)



**End of questions**

**Additional working space**

Question number: _____

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Question number: _____

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